QISKIT FALL FEST

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Circuitos Cuánticos y Primeros Algoritmos

Karen Rodríguez

Grupo de Investigación Física Teórica del Estado Sólido

Departamento de Física,

Facultad de Ciencias Naturales y Exactas

1.1 Producto tensorial

El producto tensorial entre los espacios $\mathcal{E}_1$ y $\mathcal{E}_2$ se escribe como $\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2$. Tiendo $|\alpha\rangle \in \mathcal{E}_1$ y $|\beta\rangle \in \mathcal{E}_2$, en $\mathcal{E}$ existe un ket

$$
\begin{aligned}
|\alpha\rangle \otimes |\beta\rangle &= |\alpha, \beta\rangle \\
&= |\beta\rangle \otimes |\alpha\rangle = |\beta, \alpha\rangle
\end{aligned}
$$

Advertencia: Una vez se escoje una notación se debe respetar.

Producto Tensorial

1.1.1 Propiedades:

1. Multiplicación:

   - Lineal por números complejos. Sea c_1 y $c_2 \in \mathbb{C}$ entonces:

   $$c_1|\alpha\rangle \otimes c_2|\beta\rangle = c_1 c_2|\alpha, \beta\rangle.$$

   - Distributivo respecto a la adición:

   $$|\alpha\rangle \otimes (|\beta_1\rangle + |\beta_2\rangle) = |\alpha, \beta_1\rangle + |\alpha, \beta_2\rangle.$$

Producto Tensorial

1.1.1 Propiedades:

2. Sea $\hat{A} \in \mathcal{E}_1$, su extensión a $\mathcal{E}$ es: $\hat{A} \otimes \hat{1}_2 \in \mathcal{E}$, con $\hat{1}_i \in \mathcal{E}_i$ la identidad en el espacio $\mathcal{E}_i$, donde en el presente ejemplo $i = \{1, 2\}$. De tal manera que:

$$\begin{aligned} \hat{A} \otimes \hat{1}_2 |\alpha, \beta\rangle &= \hat{A}|\alpha\rangle \otimes \hat{1}_2|\beta\rangle \\ &= \hat{A}|\alpha\rangle \otimes |\beta\rangle = \hat{A}|\alpha, \beta\rangle. \end{aligned} \tag{1.1}$$

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En general se tiene para $\hat{A} \in \mathcal{E}_1$ y $\hat{B} \in \mathcal{E}_2$ que el operador $\hat{A} \otimes \hat{B} \in \mathcal{E}$, el cual se aplica:

$$\hat{A} \otimes \hat{B}|\alpha, \beta\rangle = \hat{A}|\alpha\rangle \otimes \hat{B}|\beta\rangle$$

Producto Tensorial

1.1.1 Propiedades:

3. Producto interno: sea $|\alpha,\beta\rangle \underbrace{\longrightarrow}_{\text{D.C}} \langle\beta,\alpha|$, donde las siglas D.C son el dual correspondiente el cual es el bra, el elemento dual, del ket. Tal que:

$$\begin{aligned}\langle\beta,\alpha|\alpha,\beta\rangle &= \langle\beta| \otimes \underbrace{\langle\alpha|\alpha\rangle} \otimes |\beta\rangle \\ &= \langle\alpha|\alpha\rangle\langle\beta|\beta\rangle\end{aligned}$$

Producto Tensorial

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1.1.1 Propiedades:

4. Conjunto de kets propios: Sea $\{|a\rangle_i\}$ los kets propios del operador $A \in \mathcal{E}_1$ con $\hat{A}$ el CCOC (Conjunto Completo de Observables Compatibles) de $\mathcal{E}_1$, esto es $\hat{A}|a_i\rangle = a_i|a_i\rangle$.

Producto Tensorial

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   Y sea $\{|b\rangle_j\}$ los kets propios del operador $\hat{B} \in \mathcal{E}_2$ con $\hat{B}$ el CCOC (Conjunto Completo de Observables Compatibles) de $\mathcal{E}_2$, esto es $\hat{B}|b_j\rangle = b_j|b_j\rangle$.

Producto Tensorial

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   Entonces se arma la base en $\mathcal{E}$ como el producto tensorial de las bases de cada subespacio de kets de la forma: $\{|a_i\rangle \otimes |b_j\rangle\} = \{a_i, b_j\}$, y se tiene que:

$$\hat{A} \otimes \hat{1}_2 |a_i, b_j\rangle = \hat{A}|a_i, b_j\rangle = a_i|a_i, b_j\rangle$$
$$\hat{1}_1 \otimes \hat{B}|a_i, b_j\rangle = \hat{B}|a_i, b_j\rangle = b_j|a_i, b_j\rangle,$$

Entonces se arma la base en $\mathcal{E}$ como el producto tensorial de las bases de cada subespacio de kets de la forma: $\{|a_i\rangle \otimes |b_j\rangle\} = \{|a_i, b_j\rangle\}$ y se tiene que:

$$\hat{A} \otimes \hat{1}_2 |a_i, b_j\rangle = \hat{A}|a_i, b_j\rangle = a_i |a_i, b_j\rangle$$
$$\hat{1}_1 \otimes \hat{B} |a_i, b_j\rangle = \hat{B}|a_i, b_j\rangle = b_j |a_i, b_j\rangle,$$

Implicaciones:

(a) Debido a que $|a_i, b_j\rangle$ es ket propio simultáneo para $\hat{A}$ y $\hat{B}$, se tiene que $[\hat{A}, \hat{B}] = 0 = [\hat{A} \otimes \hat{1}_2, \hat{1}_1 \otimes \hat{B}]$, esto es, todo operador que pertenezca a $\mathcal{E}_1$ conmuta con operadores en el espacio $\mathcal{E}_2$, debido a que pertenecen a espacios Hilbert distintos.

1.1.2 Superposición

Sea $|\alpha\rangle$ un ket de estado, se puede escribir como una superposición en $\mathcal{E}_1$ usando una base completa como la que nos proporcionó $\hat{A}$ y lo mismo para $|\beta\rangle$ un ket de estado en $\mathcal{E}_2$, tal que:

$$|\alpha\rangle = \sum_i c_i |a_i\rangle \in \mathcal{E}_1 \quad \text{y} \quad |\beta\rangle = \sum_j d_j |b_j\rangle \in \mathcal{E}_2,$$

entonces se tienen dos caminos.

1.1.2 Superposición

$$|\alpha\rangle = \sum_i c_i |a_i\rangle \in \mathcal{E}_1 \quad \text{y} \quad |\beta\rangle = \sum_j d_j |b_j\rangle \in \mathcal{E}_2,$$

1. Camino 1: un ket de estado $|\eta\rangle \in \mathcal{E}$ que se define como un **estado separable** cuando:

$$\begin{aligned}
|\eta\rangle &= |\alpha\rangle \otimes |\beta\rangle \\
&= \sum_i c_i |a_i\rangle \otimes \sum_j d_j |d_j\rangle \\
&= \sum_{i,j} f_{ij} |a_i, b_j\rangle \quad \text{con} \quad f_{ij} = c_i d_j \in \mathbb{C}.
\end{aligned}$$

1.1.2 Superposición

$$|\alpha\rangle = \sum_i c_i |a_i\rangle \in \mathcal{E}_1 \quad \text{y} \quad |\beta\rangle = \sum_j d_j |b_j\rangle \in \mathcal{E}_2,$$

2. Camino 2: un ket de estado $|\gamma\rangle \in \mathcal{E}$ que se define como un **estado entrelazado** cuando:

$$\begin{aligned} |\gamma\rangle &= \alpha \otimes \beta \\ &= \sum_i c_i |a_i\rangle \otimes \sum_j d_j |d_j\rangle \\ &= \sum_{i,j} f_{ij} |a_i, b_j\rangle \quad \text{con} \quad f_{ij} \neq c_i d_j \in \mathbb{C}. \end{aligned}$$

Es un estado que NO es separable!

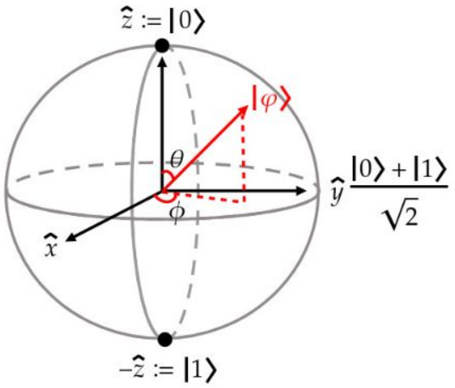
2.2 Quantum logic gates

Most common quantum gates operate on spaces of 1 or 2 qubits. The gates are represented by unitary matrices, and in general, the representation belongs to $U(2^n)$ (2^nx2^n matrices), where n is the number of qubits that the gate acts on; therefore, the state vectors have 2^n complex components.

2.2.1 Single qubit gates.

Let us start with the Pauli gates, whose representations are given by the Pauli matrices.

1. **The Pauli X-gate.**
   This gate corresponds to the classical negation (NOT) gate, and is often called the quantum NOT gate. In the Bloch sphere it corresponds to a rotation of π radians about the x-axis.



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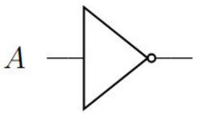
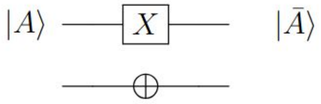


A	$\bar{A}$
0	1
1	0

Quantum version:



$\lvert A\rangle$	$\lvert \bar{A}\rangle$
$\lvert 0\rangle$	$\lvert 1\rangle$
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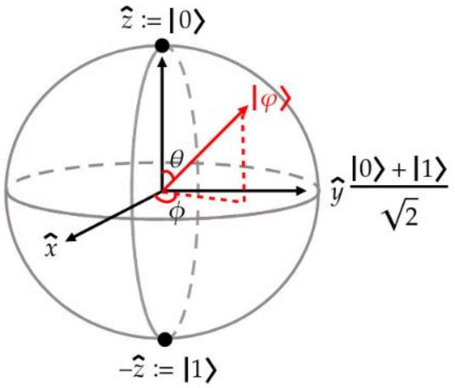

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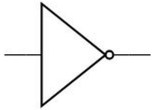

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0	1
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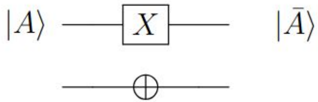
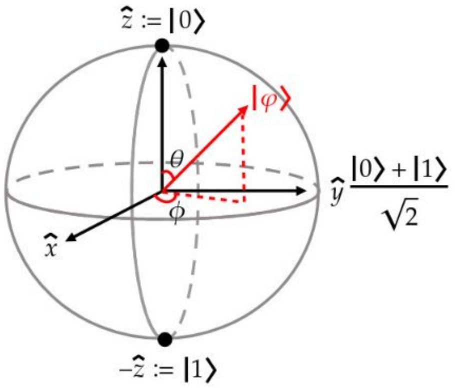


$\vert A\rangle$	$\vert \bar{A}\rangle$
$\vert 0\rangle$	$\vert 1\rangle$
$\vert 1\rangle$	$\vert 0\rangle$

Matrix representation:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



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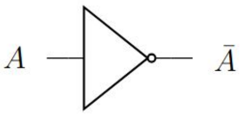	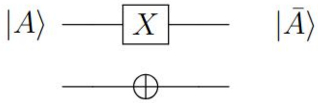

A	$\bar{A}$
0	1
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$\lvert A\rangle$	$\lvert \bar{A}\rangle$
$\lvert 0\rangle$	$\lvert 1\rangle$
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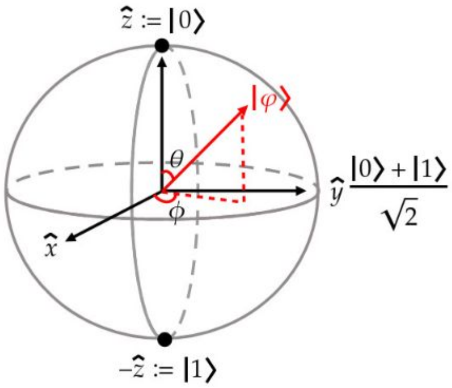
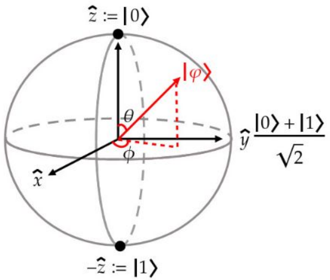
Matrix representation:

$$\underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\hat{X}} \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{|0\rangle} = \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{|1\rangle}$$





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Let us start with the Pauli gates, whose representations are given by the Pauli matrices.

1. **The Pauli X-gate.**

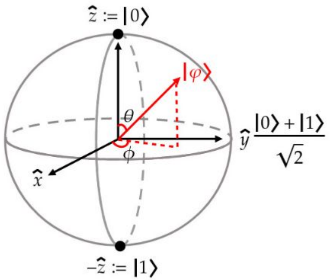
2. **The Pauli Y-gate.**
   This gate corresponds to a *NOT* gate with a phase i. In the Bloch sphere it equates to a rotation of π radians about the y-axis.

   Matrix-wise, it is $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and an operation would look like

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix}. \tag{2.1}$$

   looking in ket's space as $\hat{Y}|0\rangle = i|1\rangle$, and $\hat{Y}|1\rangle = -i|0\rangle$.

le



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Let us start with the Pauli gates, whose representations are given by the Pauli matrices.

1. **The Pauli X-gate.**
2. **The Pauli Y-gate.**
3. **The Pauli Z-gate.**
   Also known as the R_π gate, this gate represents a rotation of π radians of the Bloch sphere. It is a special case of a phase-shift gate R_ϕ, where $\phi = \pi$.

   Its matrix representation is $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

   Note that this gate leaves $|0\rangle$ unchanged, but maps $|1\rangle$ into $-|1\rangle$, hence, it is called a phase flip.

Remember that $\hat{X}^2 = \hat{Y}^2 = \hat{Z}^2 = \hat{I}$, where $\hat{I}$ is the identity operator.

2.2.1 Single qubit gates.

4. **Square root of NOT gate** ($-\boxed{\sqrt{\neg}}-$)

This gate is represented by,

$$\sqrt{X} = \sqrt{NOT} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix},$$

This way, X-gate may be recovered by,

$$X = \sqrt{X}\sqrt{X},$$

$$X = \frac{1}{4}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

2.2.1 Single qubit gates.

5. **Phase shift gate (R_ϕ).**
   The matrix representation of this general gate is given by,

$$R_\phi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}.$$

   Note that $|0\rangle$ is unchanged by the gate, but $|1\rangle \longrightarrow R_\phi|1\rangle = e^{i\phi}|1\rangle$. Although the phase of the quantum state is modified, the probability of measuring $|0\rangle$ or $|1\rangle$ is not affected.

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$$\text{Some special cases} \begin{cases} \phi = \pi\text{: Pauli Z-gate} \\ \phi = \pi/2\text{: Phase gate, sometimes written as } S. \\ \phi = \pi/4\text{: } \frac{\pi}{8}\text{-gate, written as } T. \end{cases}$$

2.2.1 Single qubit gates.

6. **Hadamard gate** (H) ($-\boxed{H}-$) This is a very useful gate, whose matrix representation is given by,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

In the ket's space, the action of this gate on the $\hat{S}_z$-basis looks like

$$|0\rangle \longrightarrow \hat{H}|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \qquad |1\rangle \longrightarrow \hat{H}|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

Universidad del Valle
Prof. Karen Rodríguez Ramírez karem.c.rodriguez@correounivalle.edu.co Gurpo de Física Teórica del Estado Sólido Departamento de Física, Univalle

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Under the action of a Hadamard gate, a measurement has an equal probability of becoming $|0\rangle$ or $|1\rangle$; the gate creates a equally-probable superposition.

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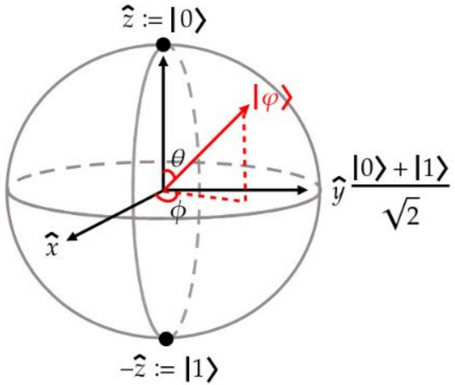
In the Bloch sphere, it corresponds to either a rotation of π radians about the $(\hat{x}+\hat{z})/\sqrt{2}$, meaning a rotation of $\pi/2$ about the x-axis followed by another one about the y-axis.



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6 **Hadamard gate (H)** (—$\boxed{H}$—)

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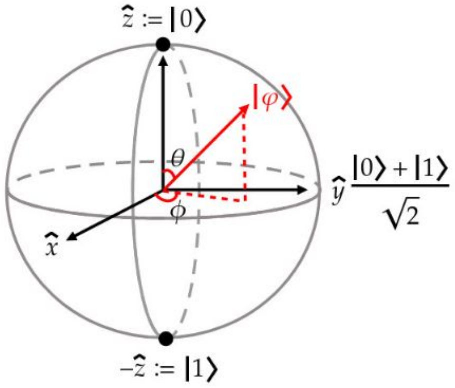
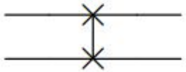


Finally, note that $\hat{H}^*\hat{H} = \hat{I}$,

and that the H-gate is the one-qubit version of the Fourier transform.

2.2.2 Two qubit gates.

All the following gates are with respect to the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

1. **Swap gate.**
   The diagram is  and the matrix representation: $$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   Its application on the basis vectors lead us to:

$$\begin{aligned} |00\rangle &\rightarrow |00\rangle, \qquad |11\rangle \rightarrow |11\rangle, \\ |01\rangle &\rightarrow |10\rangle, \qquad |10\rangle \rightarrow |01\rangle. \end{aligned}$$

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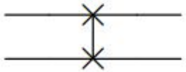
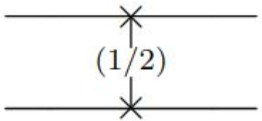
   The diagram is  and the matrix representation:

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. **Square root of swap gate.**
   The diagram is 

   and the matrix representation:

$$\sqrt{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(1+i) & \frac{1}{2}(1-i) & 0 \\ 0 & \frac{1}{2}(1-i) & \frac{1}{2}(1+i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

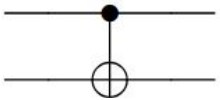
The $\sqrt{SWAP}$ gate performs a halfway two-qubit swap. It is important because any many-qubit gate can be constructed from $\sqrt{SWAP}$, and single-qubit gates.

2.2.2 Two qubit gates.

3. Controlled gates (cX, cY, cZ).

These gates act on two or more qubits, where one or more of them act as controls.

Controlled NOT gate ($CNOT$ or cX):

The diagram is  and the matrix representation: $CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$CNOT$ performs the NOT on the second qubit only when the first is $|1\rangle$, otherwise it leaves it unchanged. These controlled gates are used to generate entangled states.

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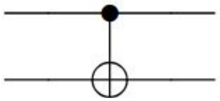
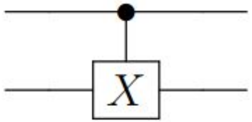
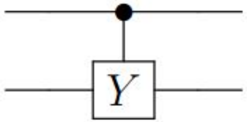
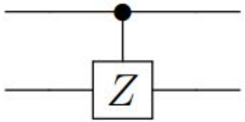
The diagram is  and the matrix representation: $CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$CNOT$ performs the NOT on the second qubit only when the first is $|1\rangle$, otherwise it leaves it unchanged. These controlled gates are used to generate entangled states.

The controlled X, Y, and, Z gates are diagrammed in a similar way:





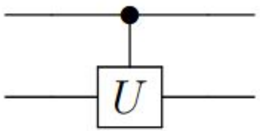


2.2.2 Two qubit gates.

3. Controlled gates (cX, cY, cZ).

These gates act on two or more qubits, where one or more of them act as controls.

In general: **controlled-U gate**

If U i a single qubit gate, for example one of the Pauli matrices, with a general expression as $U = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix}$, the diagram will be depicted by 

then,

the controlled-U matrix representation looks as follows

$$cU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & U_{00} & U_{01} \\ 0 & 0 & U_{10} & U_{11} \end{pmatrix},$$

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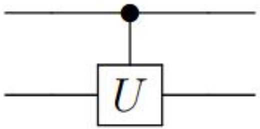

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its action on a two-qubits basis is therefore given by,

$$\begin{aligned}
|00\rangle &\rightarrow |00\rangle \\
|01\rangle &\rightarrow |01\rangle \\
|10\rangle &\rightarrow |1\rangle \otimes U|0\rangle = |1\rangle \otimes (U_{00}|0\rangle + U_{01}|1\rangle) \\
|11\rangle &\rightarrow |1\rangle \otimes U|1\rangle = |1\rangle \otimes (U_{10}|0\rangle + U_{11}|1\rangle)
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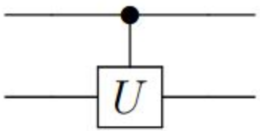

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$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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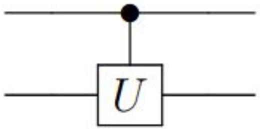

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??

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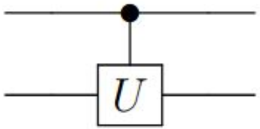

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2.2.2 Two qubit gates.

4. **Ising gate (XX).**

$$XX_\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & -ie^{i\phi} \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ -ie^{-i\phi} & 0 & 0 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & e^{i(\phi-\pi/2)} \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ e^{i(-\phi-\pi/2)} & 0 & 0 & 1 \end{pmatrix}$$

The Ising gate is implemented natively in some trapped-ion quantum computers.

2.2.3 Two qubit examples.

1. Let us construct a gate that is equal to two Hadamard gates acting in parallel:

$$G = H \otimes H =$$

G is the two-qubit Hadamard gate, and can be applied to a two-qubit vector, for example $|00\rangle$:

$$G|00\rangle =$$

We see that all states have the same probabilty.

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We see that all states have the same probabilty.

2.2.3 Two qubit examples.

2. Let us apply a single-qubit gate, like an H-gate, to a two-qubit entangled state, such as a Bell state. First, we must extend the single-qubit gate to the two-qubit domain by means of the tensor product with the identity matrix (the do-nothing gate):

$$M = H \otimes I =$$

Now, if $|v\rangle$ is a Bell state, such that $|v\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} :=$

2.2.3 Two qubit examples.

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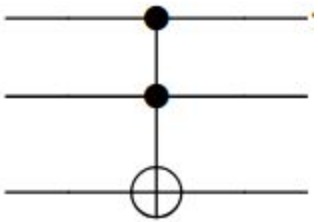
2.2.4 Three qubit gates.

For three qubit systems we will, use the basis $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$.

1. **Toffoli gate ($CCNOT$).**
   Also called the **Deutsch $D(\pi/2)$ gate**, it's a universal gate for classical computation. The quantum version works just as its classical counterpart: if the first 2 qubits are in $|1\rangle$, then it applies a Pauli X-gate (NOT) on the third one; otherwise, it does nothing.



Input	Output
000	000
001	001
010	010
011	011
100	100
101	101
110	111
111	110

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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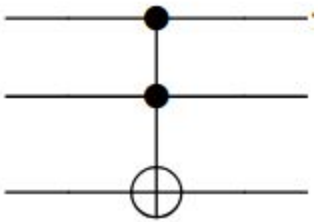

In general, the Deutsch gate $D(\theta)$ acts as follows:

$$|a,b,c\rangle \to \begin{cases} i\cos\theta|a,b,c\rangle + \sin\theta|a,b,1-c\rangle & \text{for } a=b=1, \\ |a,b,c\rangle & \text{otherwise.} \end{cases}$$


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Producto Tensorial

Recordemos!!!

1.1.1 Propiedades:

1. Multiplicación:

- Lineal por números complejos. Sea c_1 y $c_2 \in \mathbb{C}$ entonces:

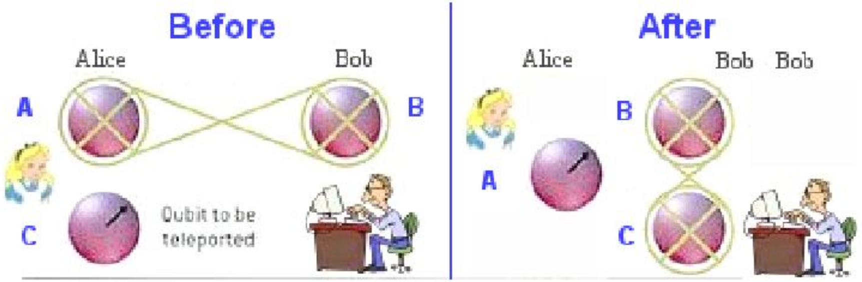
$$c_1|\alpha\rangle \otimes c_2|\beta\rangle = c_1 c_2 |\alpha, \beta\rangle.$$

- Distributivo respecto a la adición:

$$|\alpha\rangle \otimes (|\beta_1\rangle + |\beta_2\rangle) = |\alpha, \beta_1\rangle + |\alpha, \beta_2\rangle.$$

Protocolo de teleportación cuántica

Suponga que Alicia tiene el estado $|\Psi\rangle_C = \alpha|+\rangle_C + \beta|-\rangle_C$, donde $\alpha, \beta \in \mathbb{C}$, y la letra C denota el espacio de kets $\mathcal{E}_C$ que inicialmente es de Alicia, quien desea pasar la información de dicho ket a su amigo Bob, el cual vive en el espacio $\mathcal{E}_B$.

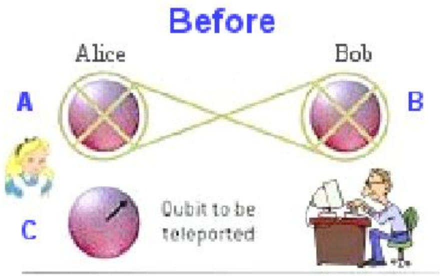


Por otro lado, Alicia y Bob comparten un estado entrelazado, esto es que no se puede separar y que vive en el espacio "agrandado" dado por el producto tensorial de ambos $\mathcal{E}_{AB}$, dado por $|\Phi_0\rangle_{AB} = \frac{1}{\sqrt{2}}[|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B]$, donde A, B denotan los espacios $\mathcal{E}_A$ y $\mathcal{E}_B$.

Protocolo de teleportación cuántica

Partimos del estado que Alicia desea teleportar:

$$|\Psi\rangle_C = \alpha|+\rangle_C + \beta|-\rangle_C,$$



Tomamos la base $\{|+\rangle, |-\rangle\}$ como base computacional efectiva, es decir,

$$|0\rangle \equiv |+\rangle, \qquad |1\rangle \equiv |-\rangle.$$

Protocolo de teleportación cuántica

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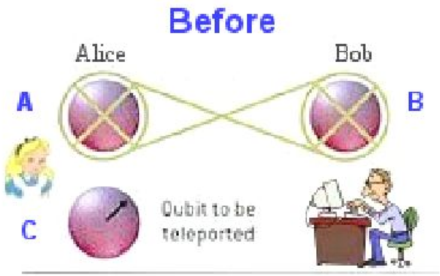

$C: \ \alpha|+\rangle + \beta|-\rangle = |\Psi\rangle_C$

$A: \ |+\rangle = |0\rangle$

$B: \ |+\rangle = |0\rangle$

Suponemos que los qubits A y B comienzan en $|+\rangle_A|+\rangle_B \equiv |0\rangle_A|0\rangle_B$.

Queremos armar el estado entrelazado que comparten Alicia y Bob:

$$|\Phi_0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B\right)$$

Preparación del par entrelazado en la base $\{|+\rangle, |-\rangle\}$

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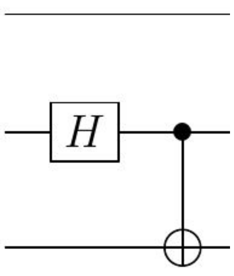
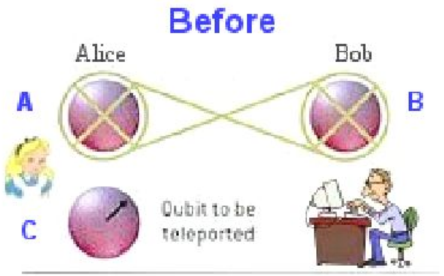




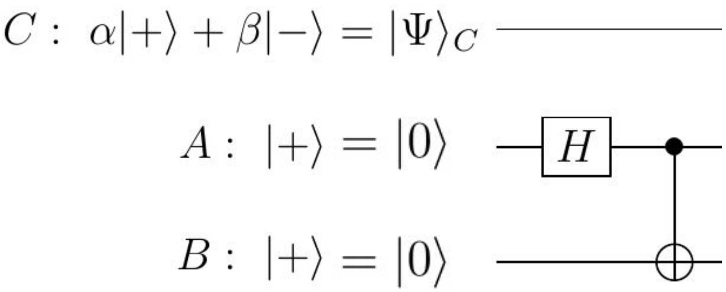
1. En la primera columna se aplica una compuerta Hadamard sobre A:

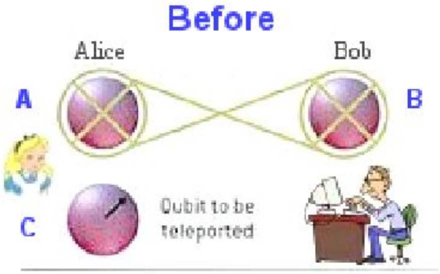
$$|+\rangle_A \xrightarrow{H} \ ?$$

2. En la segunda columna se aplica una CNOT con control en A y blanco en B:

$$\mathrm{CNOT}_{A\to B}: \ ?$$

Preparación del par entrelazado en la base $\{|+\rangle, |-\rangle\}$





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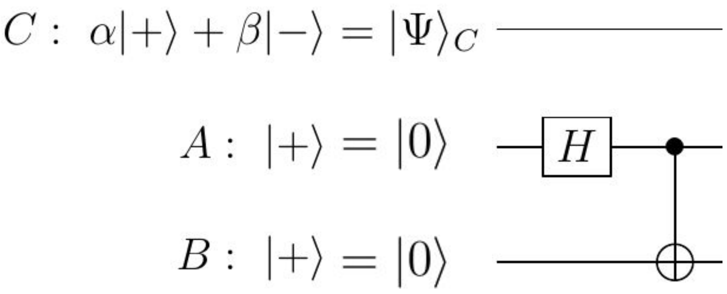
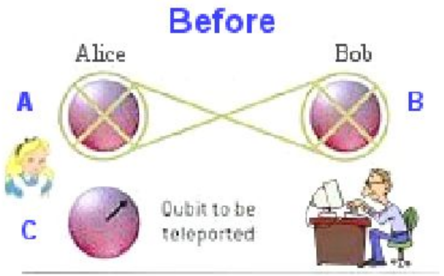
Preparación del par entrelazado en la base $\{|+\rangle, |-\rangle\}$





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2. En la segunda columna se aplica una CNOT con control en A y blanco en B:

$$\text{CNOT}_{A\to B}: \quad \frac{1}{\sqrt{2}}(|+_A +_B\rangle + |-_A -_B\rangle) = |\Phi_0\rangle_{AB},$$

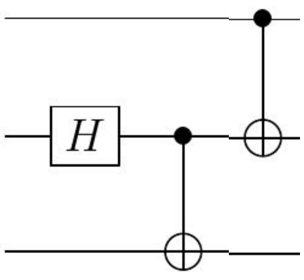
Al final de la columna 2, los qubits A y B están entrelazados en la base $\{|+\rangle, |-\rangle\}$

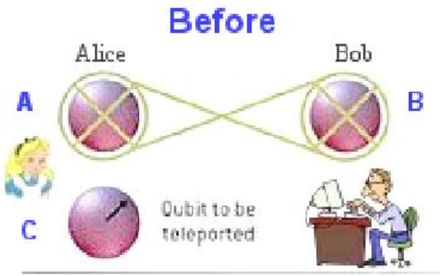
Interacción del qubit C con el par A–B

$C: \ \alpha|+\rangle + \beta|-\rangle = |\Psi\rangle_C$

$A: \ |+\rangle = |0\rangle$

$B: \ |+\rangle = |0\rangle$





El qubit C comienza en el estado desconocido $|\Psi\rangle_C = \alpha|+\rangle_C + \beta|-\rangle_C$

1. Columna 3: se aplica una CNOT con control en C y blanco en A, $\mathrm{CNOT}_{C\to A}$ que mezcla el estado desconocido de C con el par de Bell (A, B).

Producto tensorial y reorganización

$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C =$?

$\mathrm{CNOT}_{C\to A}$

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Protocolo de teleportación cuántica

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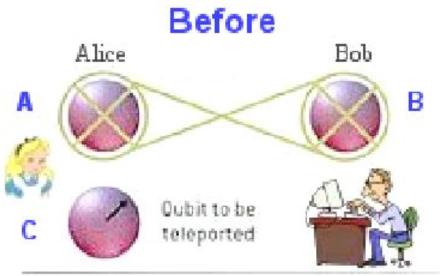
$$|\Psi\rangle_C = \alpha|+\rangle_C + \beta|-\rangle_C,$$



y del estado entrelazado que comparten Alicia y Bob:

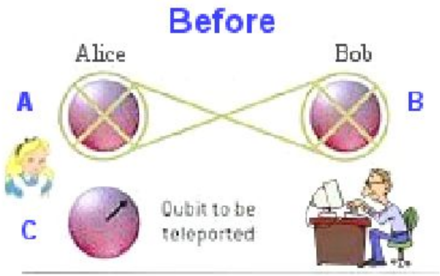
$$|\Phi_0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B\right).$$

Producto tensorial y reorganización

El estado total inicial es:

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \ ?$$

Protocolo de teleportación cuántica



Partimos del estado que Alicia desea teleportar:

$$|\Psi\rangle_C = \alpha|+\rangle_C + \beta|-\rangle_C,$$

y del estado entrelazado que comparten Alicia y Bob:

$$|\Phi_0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B\right).$$

Producto tensorial y reorganización

El estado total inicial es:

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{\sqrt{2}}\left[|+_A +_B\rangle(\alpha|+_C\rangle + \beta|-_C\rangle) + |-_A -_B\rangle(\alpha|+_C\rangle + \beta|-_C\rangle)\right]$$

Prof. Karen Rodríguez Ramírez karem.c.rodriguez@correounivalle.edu.co Gurpo de Física Teórica del Estado Sólido Departamento de Física, Univalle

Producto tensorial y reorganización

El estado total inicial es:

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{\sqrt{2}}[| +_A +_B\rangle(\alpha|+_C\rangle + \beta|-_C\rangle) + | -_A -_B\rangle(\alpha|+_C\rangle + \beta|-_C\rangle)]$$

Distribuyendo:

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{\sqrt{2}}\Big[\alpha| +_A +_B +_C\rangle + \beta| +_A +_B -_C\rangle + \alpha| -_A -_B +_C\rangle + \beta| -_A -_B -_C\rangle\Big].$$

Reordenamos como A, C, B: Alice a la izquierda - Bob a la derecha

$$|\Phi_0\rangle_{AB}\otimes|\Psi\rangle_C = \ ?$$

Producto tensorial y reorganización

El estado total inicial es:

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Distribuyendo:

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{\sqrt{2}}\Big[\alpha| +_A +_B +_C\rangle + \beta| +_A +_B -_C\rangle + \alpha| -_A -_B +_C\rangle + \beta| -_A -_B -_C\rangle\Big].$$

Reordenamos como A, C, B: Alice a la izquierda - Bob a la derecha

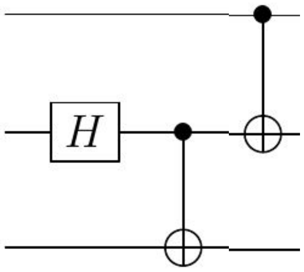
$$|\Phi_0\rangle_{AB}\otimes|\Psi\rangle_C = \frac{1}{\sqrt{2}}\left[\alpha| +_A +_C\rangle|+_B\rangle + \beta| +_A -_C\rangle|+_B\rangle + \alpha| -_A +_C\rangle|-_B\rangle + \beta| -_A -_C\rangle|-_B\rangle\right]$$

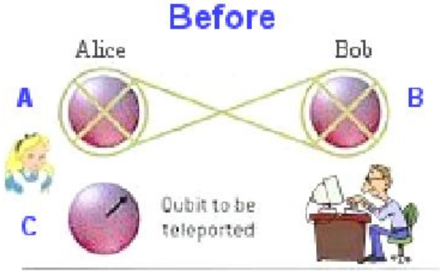
Interacción del qubit C con el par A–B

$C: \ \alpha|+\rangle + \beta|-\rangle = |\Psi\rangle_C$

$A: \ |+\rangle = |0\rangle$

$B: \ |+\rangle = |0\rangle$





El qubit C comienza en el estado desconocido $|\Psi\rangle_C = \alpha|+\rangle_C + \beta|-\rangle_C$

1. Columna 3: se aplica una CNOT con control en C y blanco en A, $\mathrm{CNOT}_{C\to A}$ que mezcla el estado desconocido de C con el par de Bell (A, B).

Reordenamos como A, C, B:

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{\sqrt{2}}\left[\alpha|+_A +_C\rangle|+_B\rangle + \beta|+_A -_C\rangle|+_B\rangle + \alpha|-_A +_C\rangle|-_B\rangle + \beta|-_A -_C\rangle|-_B\rangle\right]$$

Universidad del Valle

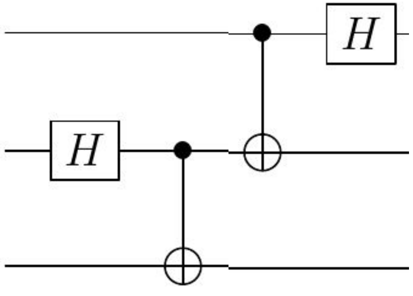
Prof. Karen Rodríguez Ramírez karem.c.rodriguez@correounivalle.edu.co Gurpo de Física Teórica del Estado Sólido Departamento de Física, Univalle

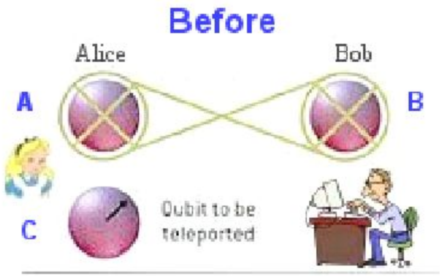
Medición de Alicia y corrección de Bob

$C: \ \alpha|+\rangle + \beta|-\rangle = |\Psi\rangle_C$

$A: \ |+\rangle = |0\rangle$

$B: \ |+\rangle = |0\rangle$





2. Columna 4: se aplica una compuerta Hadamard sobre C, $\hat{H}_C$, que completa la transformación que, a nivel algebráico, lleva el estado global a la descomposición

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{2}\sum_{i=0}^{3} |\Phi_i\rangle_{AC} \otimes (\text{corrección}_i |\Psi\rangle_B),$$

donde los estados de Bell $|\Phi_i\rangle_{AC}$ están definidos en la base $\{|+\rangle, |-\rangle\}$.

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2.2.2 Two qubit gates.

Recordemos!!!

All the following gates are with respect to the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

Definimos los cuatro estados de Bell, son los cuatro posibles estados entrelazados

$$|\Phi_0\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|+_A +_C\rangle + |-_A -_C\rangle\right),$$

$$|\Phi_1\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|+_A -_C\rangle + |-_A +_C\rangle\right),$$

$$|\Phi_2\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|+_A -_C\rangle - |-_A +_C\rangle\right),$$

$$|\Phi_3\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|+_A +_C\rangle - |-_A -_C\rangle\right).$$

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$$|\Phi_3\rangle_{AC} = \frac{1}{\sqrt{2}}\left(|+_A +_C\rangle - |-_A -_C\rangle\right).$$

Y sus identidades inversas:

$$|+_A +_C\rangle = \tfrac{1}{\sqrt{2}}\left(|\Phi_0\rangle_{AC} + |\Phi_3\rangle_{AC}\right),$$

$$|-_A -_C\rangle = \tfrac{1}{\sqrt{2}}\left(|\Phi_0\rangle_{AC} - |\Phi_3\rangle_{AC}\right),$$

$$|+_A -_C\rangle = \tfrac{1}{\sqrt{2}}\left(|\Phi_1\rangle_{AC} + |\Phi_2\rangle_{AC}\right),$$

$$|-_A +_C\rangle = \tfrac{1}{\sqrt{2}}\left(|\Phi_1\rangle_{AC} - |\Phi_2\rangle_{AC}\right).$$

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Protocolo de teleportación cuántica

Reordenamos como A, C, B:

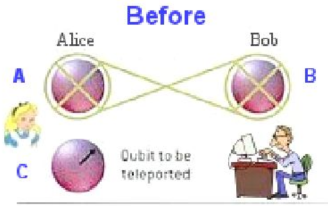
$$|\Phi_0\rangle_{AB}\otimes|\Psi\rangle_C = \frac{1}{\sqrt{2}}\left[\alpha|+_A +_C\rangle|+_B\rangle + \beta|+_A -_C\rangle|+_B\rangle + \alpha|-_A +_C\rangle|-_B\rangle + \beta|-_A -_C\rangle|-_B\rangle\right]$$

Sustituyendo:

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \quad ?$$

Y sus identidades inversas:

$$\begin{aligned}
|+_A +_C\rangle &= \tfrac{1}{\sqrt{2}}\left(|\Phi_0\rangle_{AC} + |\Phi_3\rangle_{AC}\right),\\
|-_A -_C\rangle &= \tfrac{1}{\sqrt{2}}\left(|\Phi_0\rangle_{AC} - |\Phi_3\rangle_{AC}\right),\\
|+_A -_C\rangle &= \tfrac{1}{\sqrt{2}}\left(|\Phi_1\rangle_{AC} + |\Phi_2\rangle_{AC}\right),\\
|-_A +_C\rangle &= \tfrac{1}{\sqrt{2}}\left(|\Phi_1\rangle_{AC} - |\Phi_2\rangle_{AC}\right).
\end{aligned}$$



Alice a la izquierda - Bob a la derecha

Protocolo de teleportación cuántica

Reordenamos como A, C, B:

$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{\sqrt{2}}\left[\alpha|+_A +_C\rangle|+_B\rangle + \beta|+_A -_C\rangle|+_B\rangle + \alpha|-_A +_C\rangle|-_B\rangle + \beta|-_A -_C\rangle|-_B\rangle\right]$$

Sustituyendo:

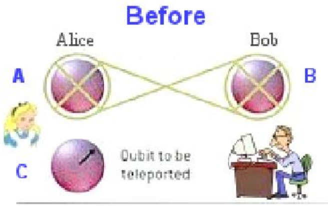
$$\begin{aligned}|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{2}\Big[&|\Phi_0\rangle_{AC}(\alpha|+_B\rangle + \beta|-_B\rangle) + |\Phi_1\rangle_{AC}(\beta|+_B\rangle + \alpha|-_B\rangle)\\ &+ |\Phi_2\rangle_{AC}(\beta|+_B\rangle - \alpha|-_B\rangle) + |\Phi_3\rangle_{AC}(\alpha|+_B\rangle - \beta|-_B\rangle)\Big].\end{aligned}$$

Esta es la forma pedida: cada sumando contiene

$$|\Phi_i\rangle_{AC} \otimes (\text{estado en } B).$$

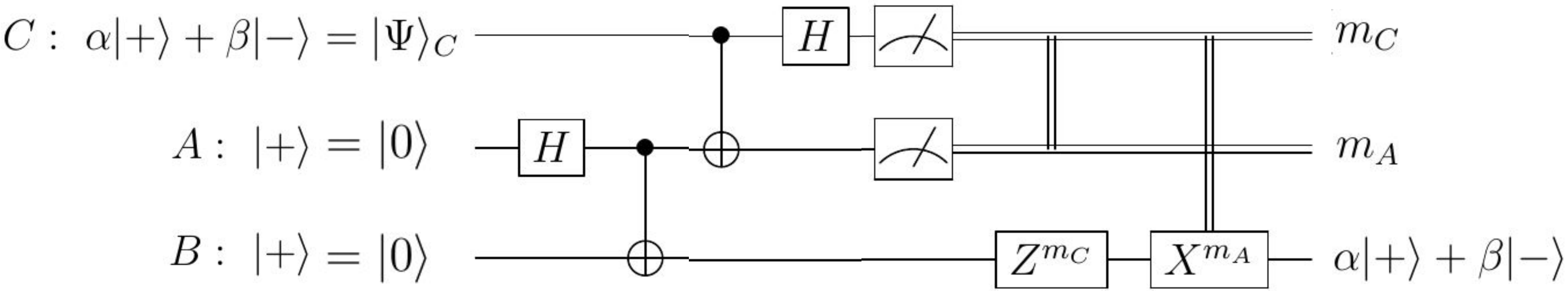
Y sus identidades inversas:

$$\begin{aligned}|+_A +_C\rangle &= \tfrac{1}{\sqrt{2}}\left(|\Phi_0\rangle_{AC} + |\Phi_3\rangle_{AC}\right),\\ |-_A -_C\rangle &= \tfrac{1}{\sqrt{2}}\left(|\Phi_0\rangle_{AC} - |\Phi_3\rangle_{AC}\right),\\ |+_A -_C\rangle &= \tfrac{1}{\sqrt{2}}\left(|\Phi_1\rangle_{AC} + |\Phi_2\rangle_{AC}\right),\\ |-_A +_C\rangle &= \tfrac{1}{\sqrt{2}}\left(|\Phi_1\rangle_{AC} - |\Phi_2\rangle_{AC}\right).\end{aligned}$$

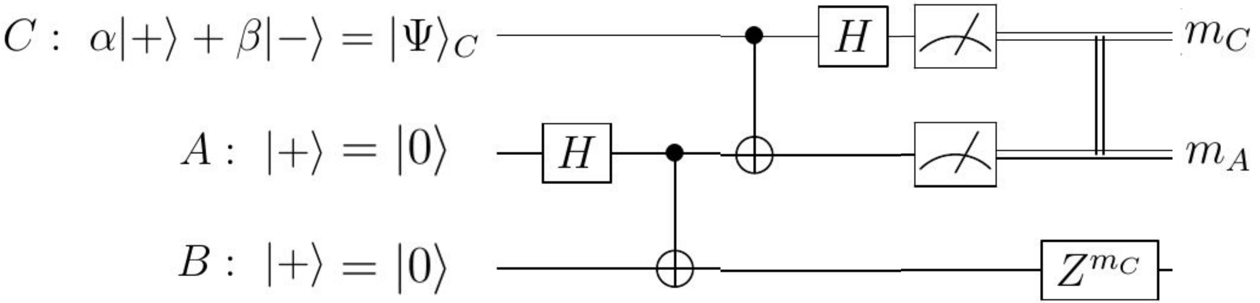


Alice a la izquierda - Bob a la derecha

Medición de Alicia y corrección de Bob



Medición de Alicia y comunicación clásica.



En la columna 5, Alicia mide C y A en la base $\{|+\rangle, |-\rangle\}$, obteniendo dos bits clásicos (m_C, m_A).

El estado de Bob colapsa a una de las versiones rotadas de $\hat{Z}^{m_C}|\Psi\rangle_B$:

$$|\Psi\rangle_B, \quad \hat{\sigma}_x|\Psi\rangle_B, \quad \hat{\sigma}_y|\Psi\rangle_B, \quad \hat{\sigma}_z|\Psi\rangle_B.$$

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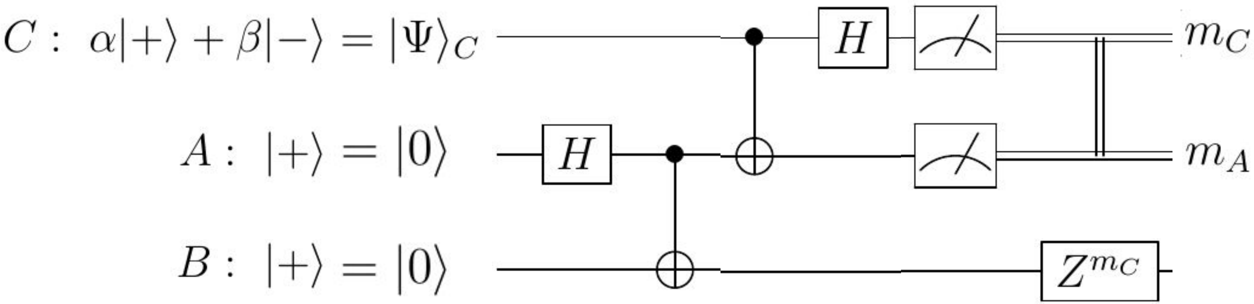
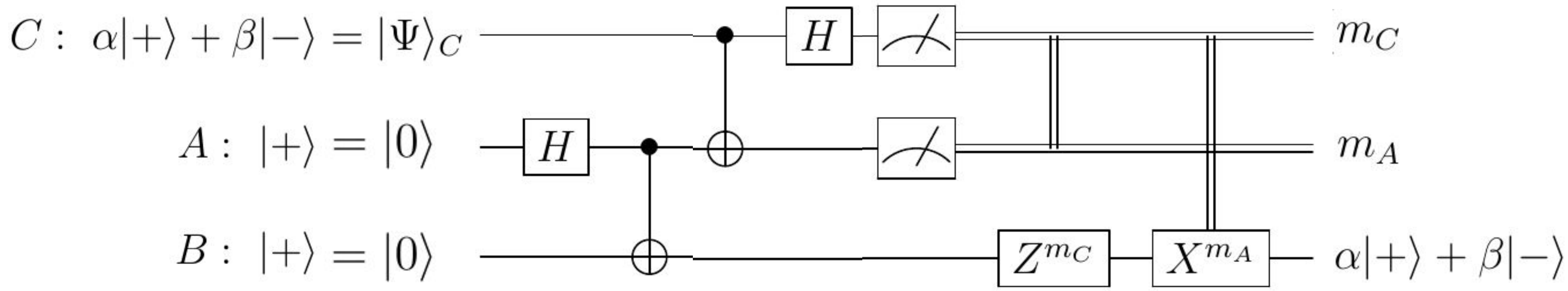
Medición de Alicia y comunicación clásica.



$$|\Phi_0\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{2}\Big[|\Phi_0\rangle_{AC}(\alpha|+_B\rangle + \beta|-_B\rangle) + |\Phi_1\rangle_{AC}(\beta|+_B\rangle + \alpha|-_B\rangle)$$
$$+ |\Phi_2\rangle_{AC}(\beta|+_B\rangle - \alpha|-_B\rangle) + |\Phi_3\rangle_{AC}(\alpha|+_B\rangle - \beta|-_B\rangle)\Big].$$

- Si Alicia proyecta en $|\Phi_0\rangle_{AC}$, Bob tiene $|\Psi\rangle_B$;
- Si proyecta en $|\Phi_1\rangle_{AC}$, Bob tiene $\hat{\sigma}_1|\Psi\rangle_B$;
- Si proyecta en $|\Phi_2\rangle_{AC}$, Bob tiene $\hat{\sigma}_2|\Psi\rangle_B$ (hasta una fase global);
- Si proyecta en $|\Phi_3\rangle_{AC}$, Bob tiene $\hat{\sigma}_3|\Psi\rangle_B$.

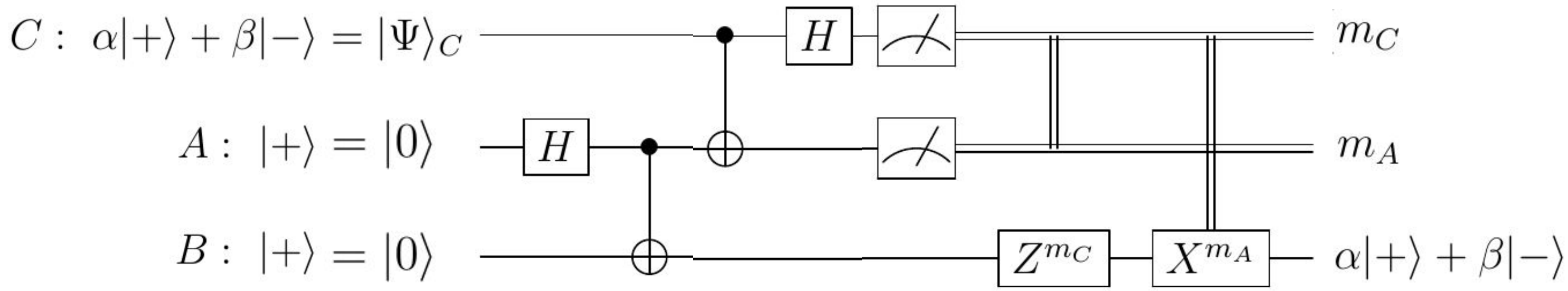
Corrección local de Bob.



En la columna 6, Bob recibe los dos bits clásicos y aplica en su qubit la compuerta $\hat{Z}^{m_C}\hat{X}^{m_A}$, lo que, en todos los casos, reconstruye el estado original:

$$\hat{Z}^{m_C}\hat{X}^{m_A}|\text{estado colapsado}\rangle = |\Psi\rangle_B = \alpha|+_B\rangle + \beta|-_B\rangle.$$

Corrección local de Bob.



En las columnas finales del circuito, Bob aplica, de manera condicional a estos bits clásicos, compuertas de Pauli en su qubit B, $Z^{m_C}X^{m_A}$. Es decir:

(m_C, m_A)	Compuerta en B
$(0,0)$	$\mathbb{I}$
$(0,1)$	$\hat{X}$
$(1,0)$	$\hat{Z}$
$(1,1)$	$\hat{Z}\hat{X}$

$$\hat{Z}^{m_C}\hat{X}^{m_A}|\text{estado colapsado}\rangle = |\Psi\rangle_B = \alpha|+_B\rangle + \beta|-_B\rangle$$

Karem Rodríguez

Profa. Departamento de Física
Facultad de Ciencias Naturales y Exactas
Universidad del Valle

Correo

karem.c.rodriguez@correounivalle.edu.co